Don't just stand there. Do something! (about school mathematics that is)

by Nicholas L. Pappas, Ph.D.[1]

Some time ago my attention was directed to what was going on in schools when my oldest son asked me to explain "sets." I was able to respond, because I use mathematics everyday in my electronic circuit design work. That question was my introduction to the "new math" of the 60's. And so my journey into the world of school mathematics began. In the meantime I answered all questions from my gang of four, while volunteering information my experience had taught me was important for them to know and understand.

Interactions with teachers and principles at my children's schools rapidly created the perception in my mind "new math" was the current in thing to do, the latest fad. Furthermore, most of them did not know what I was talking about when I inquired about the Standard Arithmetic Algorithms for calculating with integers. Allow me to explain.

Algorithms There are four Standard Arithmetic Algorithms recognized throughout the world! One each for addition, multiplication, subtraction, and division of integers. Perhaps I should say that an algorithm is a procedure with precise instructions, requiring **no** creative skills of the user, specifying a finite number of steps so that sooner or later the procedure ends (e.g. see the Appendix).

The genius, the extraordinary innovations, of the four algorithms is that any N-digit problem is converted into N one-digit problems, which students can solve in their heads. For example, division of the 4-digit number 1254 by 7 is converted into 4 one-digit problems 1/7, 12/7, 55/7, and 64/7 where slash / represents "divided by." What I just said is for-sure-definitely *not obvious* until you execute the division algorithm which allows you to divide and conquer! No pun intended. Know that students are supposed to, and can, solve the one-digit division problems in their heads. Can do, because they know the addition and multiplication tables. Let me back up.

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NLP has worked continuously in the electronics industry since 1950 except for 11 semesters teaching at San Jose State University (1988-1993) in order to fulfill his self imposed obligation for public service. He attended K-12 in the Manhasset, NY school system (a fabulous stroke of luck). After the war he attended Lehigh University, and then transferred to Stanford where he earned the MS degree and, while working at HP in the 50's, the Ph.D (EE) degree. He never applied for the BS. His current activity is designing integrated circuits.

The first skill to learn is how to count. The first rule is that only symbols 0 to 9 are allowed. Wouldn't you like to be given $5,632,789? A many digit number using only the symbols 0 to 9. Clearly the Romans blew it big time. The number following 9 is 9+1, which is how we create any next number, add 1. To shorten a long story the world said use more digits and 10 (one, zero) was born. then on to 99, 99+1 and 100 was born. Then on to infinity which is an alias here for a number as large as we please. "We can count" means we know how to write any number we please. We learn the rules are the digit to the left has tens times the weight, only one digit is allowed in any position, all digits must be positive, plus several more. A small number of ideas is involved.

The world took it from there and proceeded to calculate. Got to add up the money, right? Addition was invented to count efficiently. I prefer to add 52984 to 67930. I do not want to count from 67930 by adding one 52984 times. The Standard Addition Algorithm converts the 5 digit addition problem just mentioned into five one-digit addition problems, because *the idea of carry* is used. Briefly put, carry allows you to add each column of digits independently so that one problem becomes five problems. One-digit problems you can do in your head, because you know the addition table. The same can be said for Standard Subtraction Algorithm where the *borrow idea* replaces the carry idea.

There is a neat, instructive way to create the multiplication table of the 100 products of digits 0 to 9 ranging from 0×0 to 9×9, which I have not seen in any Arithmetic text. This is level 0. At the next level (level 1) the carry idea facilitates conversion of 5×9361 into four one-digit problems, $5 \times 1 + 0$, $5 \times 6 + 0$, $5 \times 3 + 3$, $5 \times 9 + 1$, which students can also do in their heads one at a time. Moving up to level 2 the same ideas are used again to multiply a many digit number by a many digit number, such as 375×9361, by using the level 1 procedure three times to calculate the partial products 3×9361, 7×9361, and 5×9361. The final step is properly positioning the partial products and then adding.

Algorithms are used, because they avoid reinventing the wheel. There is no need to reinvent the algorithm every time we want to add, mul, sub or div two numbers. Critics say that's rote memory. Pay no attention to the critics, because it is not rote memory, and algorithms save us time so that we focus on non routine activities.

Math school books have many pages devoted to problems. Teachers supplement these problems with pages they create. This energy can be devoted to teaching if *arithmetic software programs* are available that implement the four standard algorithms. Then you have an unlimited number of problems available. I have written programs that "show" you how to add, mul, sub, or div, as you press Enter and follow the cursor to the next step. By the way using the programs is straightforward. The learning time is very short. You can find programs on the web, but I found only one that tries to "show". None offer a text.

A competent, serious, professional will tell you that acquiring knowledge is hard work. Specifically, many mathematicians will tell you that, even for them, learning and understanding mathematics at any level is not easy! Surprised? Furthermore acquiring knowledge of Arithmetic that ***we do not dumb down***, can be a tough, difficult, and rewarding work effort. A work effort that will enable you to function at a higher intellectual level. Learning mathematics does that for you.

Let me change course. My practice is to document my projects as I proceed. The writings expose lack of knowledge, gaps in logic, errors, and so forth. Writings are a positive response to the reality "If you cannot write it up in plain English, you do not understand it." A few years ago I put myself to the "Do I know how to explain school math?" test. Before I started writing what has evolved into unpublished *Arithmetic - Integers, Fractions, Decimals* I went to the library, because I did not want to reinvent the wheel. There I did not find any writings that presented the theory of school arithmetic! Perhaps I missed one? Essentially all the writings I found at the Library of Congress, where they have 'every' book, boiled down to "if you want to add, mul, sub, or div do it this way." I started writing.

The omission of theory in these books is a very sad state of affairs, because knowledge of the theory allows you to be flexible when you are faced with a problem you never saw before. Without the understanding that flows from this knowledge you find yourself in the box of rote that inhibits moving on to higher math. Mathematics is cumulative. The prerequisites accumulate as you move to the next topic of study.

On the Internet my first discovery was that special interests have moved the teaching of mathematics into the political arena. Children deserve better than being pawns in the pursuit of personal agendas. Agendas which invariably are about money and/or power, and absolutely-for-sure-definitely not about the interests of the children. The New York City school mess is, hopefully was, a perfect example.

An egregious example is provided by special interests who do not want to teach the standard algorithms, because they want the students to "invent" their own algorithms! Then there are special interests who want to ban the teaching of the division algorithm. Then I discovered the people who are preaching the dumbing down of the teaching of Arithmetic. Specifically some were preaching "do not teach the division algorithm, its too hard". And, none of the preachers mentioned the theory of arithmetic. They did not have to, because it is not in the curriculum. Can you guess why? The special interests give serious mathematicians heartburn.

Not providing the students the benefit of the genius of the standard algorithms is a form of intellectual deprivation. Briefly, here is why. You study the Standard Addition and Multiplication Algorithms to prepare for the Standard Division Algorithm, which has immediate K-12 applications in the study of fractions, the conversion of fractions to decimals, Euclid's greatest common divisor algorithm, and the division with remainder of algebraic polynomials. The division algorithm is also a crucial element of the foundation of many topics in University mathematics such as number theory.

Most likely those who pressure to dumb down presentations never had to raise any children. Any parent knows how smart, how curious children are. Remember the first WHY? Better yet, the first WHY NOT? Enough said?

So why all this stupidity, this nonsense? Who are these people? What are their mathematical qualifications? Why do they have influence? I recommend paying absolutely NO attention to them.

I did find serious mathematicians objecting to the dumbing down, objecting to tests that test nothing relevant, objecting to teaching to the tests. On the other hand they are patiently trying to explain the ideas and the algorithms.

Difficult? I mentioned those who want to dumb down, because math is difficult. Well it is. Difficult perhaps, but not impossible as some would have you believe. “Math is difficult” is an often repeated statement whose implications are simply misleading. A statement that is a disservice to anyone who is turned away from math by it, because knowing math gives you a highly competitive advantage, a real edge. Please consider why *learning mathematics is not different nor more difficult than learning to read.*

First of all *anything* is hard if an effort is required to learn about it. Don’t pick on math. For example, most of us have forgotten the long and hard effort required to learn to read. Today we just read. Nothing to it. Right? Really?

Reading a novel or newspaper is not difficult, because there is nothing else to learn except for a new word or two -- you just read. The only symbols used are the letters of the alphabet, punctuation, and, perhaps one or two numbers. This is why reading a page may take only a few minutes -- what you see on each page is familiar. However if you do not know the meaning of specific words you may stop to look them up in a dictionary, or you simply gloss over them with essentially *no negative consequences*.

On the other hand reading mathematics appears to be significantly different. In addition to the alphabet there are lots of numbers, many new words, strange symbols, drawings, and perplexing combinations of letters, numbers and other symbols. The mathematics reader has to learn many things at the same time such as vocabulary, symbols, and how to manipulate mathematical expressions. Furthermore, glossing over has serious negative consequences. This complexity creates the perception mathematics is difficult. As we have said, it is. Nevertheless, let us be fair.

Accurate recall will remind you that learning to read did not happen overnight (how many years was it?). In the beginning you had to learn the alphabet symbols, the sound for each symbol, how to pronounce combinations of symbols (words), and on and on. It was not easy. However you learned by doing. The more you read the better you could read. *In this sense learning mathematics is not different nor more difficult than learning to read.*

Nothing new By the way there is *nothing new* about the teaching-of-math problem. Frank H. Hall wrote the monograph "Arithmetic: How to Teach it." This was published in 1900, repeat 1900. Mr. Hall's excerpts from the Report of the Committee of Ten:

> The "Committee of Ten" was appointed at the meeting of the National Education Association in Saratoga in July 1892. Its chairman was Charles W. Eliot, President of Harvard University.
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> The conference [on Mathematics] consisted of one government official, a university professor, five professors of mathematics in as many colleges, two teachers of mathematics in endowed schools, and one proprietor of a private school for boys. The professional experience of these gentlemen and their several fields of work were various, and they came from widely separated parts of the country; yet they were unanimously of opinion that **a radical change in the teaching of arithmetic was necessary** (my bold)

Reference: Hall, Frank H. Arithmetic: How to Teach it. New York, Chicago, Werner School Book Company, 1900. p. cm. LC CALL NUMBER: QA135 .H2

Must know You stand in front of a class, present information, hopefully (that's right) elicit questions, and answer same. That's about it. Student acceptance depends on many considerations. I have been told by students that the number one consideration is "does teacher know what he/she is talking about." Given that students are students, the same must be true in K-12 schools. In other words I do not know of a substitute for knowledge of the subject.

What needs to be done? Anyone who has thought about presenting mathematics knows we need competent textbooks presenting the subject effectively each year from K to 12, as well as an adequate supply of teachers who KNOW mathematics.

Publishers need to include the theory of arithmetic in their beautiful, expensive, "800 page" four color productions.

The supply of teachers exists. They show up for work every day. We simply have to put information in their hands so that they can self study and come up to speed with school math (134 pages plus index does the job).

The Standard Division Algorithm

Division by Numbers with any number of digits

Experienced readers are asked to pretend this is the first time they are reading about this topic. Given your experience you divide quickly, because you have mastered the process. You just do it like you read. Zoom! Nevertheless pretend. Thank you.

We recommand you focus on why the algorithm is written the way it is, and why the correct answer is always produced. The latter is definitely not obvious. Hints: Since division is an efficient way to subtract many copies of the same number, the divisor, what does each subtraction in the algorithm represent?

Addition and multiplication algorithms start with creation of tables involving one digit numbers 0 to 9, and build from there. There are no division tables to build upon. The standard division algorithm requires you to do division problems such as 45/23 in your head, which arises from the digit 5 in the dividend 45224. Use the multiplication table to help you solve these elementary problems. The divisor 23 has two digits. Is anyone surprised when the standard algorithm proceeds in the same way when the divisor has any number of digits? (Try 3479021/437.) Here is an example of the division algorithm.

45224/23 Dividend$_1$ is 45224. The divisor is 23 in every step.. The equation used in every step is *m=(q × d)+r, which means dividend=(quotient × divisor) + remainder*. The steps from step 2 to the end are identical procedures.

Step 1: Execute a one digit division where the dividend is the leftmost digit 4 of dividend$_1$ = 45224.

$$\begin{array}{r} \mathbf{0} \\ \mathbf{23\overline{)45224}} \\ \mathbf{-0} \\ \hline \mathbf{4} \end{array}$$

Divide 4 by 23. $4 = (0 \times 23) + 4$ $[m=(q \times d)+r]$
Enter the quotient 0 over the leftmost digit 4, subtract 0×23=0 leaving remainder r=4.

Step 2: Multiply the remainder 4 of the preceding step by 10 and add to it the next digit 5 in dividend$_1$ to produce dividend$_2$.

$$\begin{array}{r} \mathbf{01} \\ \mathbf{23\overline{)45224}} \\ \mathbf{-0} \\ \hline \mathbf{45} \\ \mathbf{-23} \\ \hline \mathbf{22} \end{array}$$

Dividend$_2$ = $(4 \times 10) + 5 = 45$.
Divide 45 by 23 $45 = (1 \times 23) + 22$
Enter the quotient 1 over the next digit 5, subtract 1×23=23 leaving remainder 22.

Step 3: Multiply the remainder 22 of the preceding step by 10 and add to it the next digit 2 in dividend_1 to produce dividend_3.

$\text{Dividend}_3 = (22 \times 10) + 2 = 222.$
Divide 222 by 23 $\quad 222 = (9 \times 23) + 15$
Enter the quotient 9 over the next digit 2, subtract $9{\times}23{=}207$ leaving remainder 15.

$$\begin{array}{r} 019 \\ 23\overline{)45224} \\ -0 \\ \hline 45 \\ -23 \\ \hline 222 \\ -207 \\ \hline 15 \end{array}$$

Step 4: Multiply the remainder 15 of the preceding step by 10 and add to it the next digit 2 in dividend_1 to produce dividend_4.

$\text{Dividend}_4 = (15 \times 10) + 2 = 152.$
Divide 152 by 23 $\quad 152 = (6 \times 23) + 14$
Enter the quotient 6 over the next digit 2, subtract $6{\times}23{=}138$ leaving remainder 14.

$$\begin{array}{r} 0196 \\ 23\overline{)45224} \\ -0 \\ \hline 45 \\ -23 \\ \hline 222 \\ -207 \\ \hline 152 \\ -138 \\ \hline 14 \end{array}$$

Step 5: Multiply the remainder 14 of the preceding step by 10 and add to it the next digit 4 in dividend_1 to produce dividend_5.

$\text{Dividend}_5 = (14 \times 10) + 4 = 144.$
Divide 144 by 23 $\quad 144 = (6 \times 23) + 6$
Enter the quotient 6 over the next digit 4, subtract $6{\times}23{=}138$ leaving remainder 6.

Since 6 is greater than 0 and less than 23 we are done.

$$\begin{array}{r} 01966 \\ 23\overline{)45224} \\ -0 \\ \hline 45 \\ -23 \\ \hline 222 \\ -207 \\ \hline 152 \\ -138 \\ \hline 144 \\ -138 \\ \hline 6 \end{array}$$

Recap
$004 = (0 \times 23) + 4$
$045 = (1 \times 23) + 22$
$222 = (9 \times 23) + 15$
$152 = (6 \times 23) + 14$
$144 = (6 \times 23) + 6$

Answer $\quad \mathbf{45224 = 1966 \times 23 + 6}$
Observe that the quotient 01966 can be perceived by reading vertically down the first column of digits on the right side of equals. The final remainder is 6 (the last term of the last equation). We ask whether or not you consider this to be important for K-12 students to learn and know.