“Curriculum Makes a Huge Difference” - A Summary of Conclusions from the Trends in International Mathematics Study (TIMSS) with California Data Added

Derived from “A Coherent Curriculum, The Case of Mathematics”, W. Schmidt, R. Houang and L. Cogan, American Educator, Summer 2002 Issue

William Hook, March 5, 2004

Introduction and Summary: The director and senior staff of the U.S. National Research Center for the TIMSS at Michigan State University have published a paper detailing their conclusions regarding why the U.S. performance is so much worse than that of the six leading foreign countries [1]. They also present conclusions as to why the gap between the U.S. and the foreign countries continually widens as children proceed through the higher grades. All this is based on a long and detailed study of the 1995 TIMSS data.

They found that differences in achievement are related to what is taught. It is not primarily a matter of demographics or other non-school issues. Specifically *the curriculum itself - what is taught – makes a huge difference*. They characterize the U.S. *intended content* in four ways:

- Not focused (far too many topics, particularly in the lower grades)
- Highly repetitive (topics introduced too early, too little depth, endlessly repeated)
- Not very demanding (especially in middle school years)
- Incoherent (not presented in logical, step-by-step order)

They discuss the ever-widening test score gap, as a function of the school grade level, between the U.S. and the foreign countries. They note this gap is mirrored by the same ever-widening gap between the children of well-off or sophisticated parents and those of the disadvantaged within the U.S. They assert “a systemic failure to teach *all* the children the knowledge they need in order to understand what the next grade has to offer is the major source of avoidable injustice in our schools”. It is the early grades where the damage is done, and it is the early grades which must be fixed.

The purpose of this paper is to summarize their basic data in graphical form, and to add the California data.

What is actually taught: The researchers defined the “intended content” as that which comprises the national curriculum or state standards, and found that the intended content is essentially replicated in the textbooks, and that teachers “follow” the textbooks, guided by depth and duration of each topic in the textbook..

> **“we can say with statistical confidence that what is stated in the “intended content”, and in the textbooks is, by and large, taught in the classrooms of most TIMSS countries.”**

The above statement is one major conclusion from the TIMSS curriculum study.

Intended Content of the Top Six Countries Compared to U.S.: The top six achieving TIMSS countries were designated the A+ countries. An extensive list of math topics (Appendix A) was given to education officials and researchers in each of the A+ countries, and the topics intended for each country, by grade, were compiled, based on the national curriculum (intended content).

Figure 1 shows the topics which were intended by at least 2/3 of the A+ countries, as well as the average number of additional intended topics not meeting the 2/3 criteria. The same list was used by researchers to evaluate the math standards of the 21 participating U.S. states.

Figure 2 shows the same information for the U.S., indicating the intended topics for at least 2/3 of the states participating, plus the average number of additional intended topics not meeting the 2/3 criteria. All the topics to the left of the red dotted line are not taught in the A+ countries at the indicated grade. It can be seen there are far more topics taught in the U.S, particularly in the early grades. Figures 1 and 2 have the same information as Figures 1 and 2 of the Schmidt article.

Figure 3 shows the total number of topics taught in the A+ countries by grade, compared to the total number of topics taught in the U.S. by grade. It can be seen that the number of topics taught in the U.S. in the critical early grades ranges from over four times as many in grade 1 to over twice as many in grade 3.

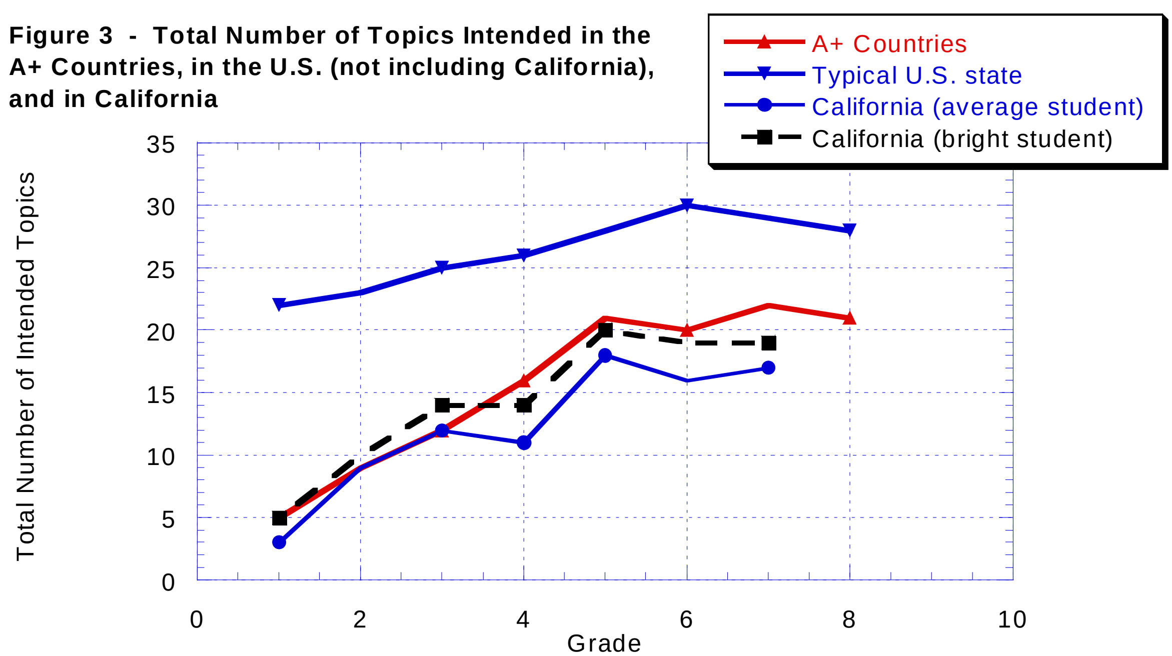

Figure 3. Showing the total number of intended topics from the TIMSS curriculum study for the A+ countries, the typical U.S. state, and for California. The graph for the California average student is based on the California Key Standards, is the core curriculum set forth in the California Framework Document, and accounts for a minimum of 70% of the questions on the yearly California Standards Test (CST). The graph for the bright student is based on a number of additional non-Key standards, as shown on the bottom of Figure 4.

Figure 1 – A+ Composite: Mathematics Topics Intended at Each Grade by At Least Two-thirds of A+ Countries

Topic	Grade 1	2	3	4	5	6	7	8
Whole Number Meaning	■	■	■	▣	▣			
Whole Number Operations	■	■	■	■	▣			
Measurement Units	□	■	■	■	■	□	▣	
Common Fractions			□	■	■	▣		
Equations and Formulas			□	▣	▣	▣	■	■
Data Representation & Analysis (Graphing)			□	□	▣	▣		□
2-D Geometry: Basics			□	▣	▣	▣	■	■
Polygons & Circles				▣	▣	▣	■	■
Perimeter, Area & Volume				▣	▣	▣	▣	□
Rounding & Significant Figures				▣	▣			
Estimating Computations				▣	▣	▣		
Properties of Whole Number Operations				□	▣			
Estimating Quantity & Size				□	□			
Decimal Fractions				▣	■	▣		
Relationship of Common & Decimal Fractions				▣	■	▣		
Properties of Common & Decimal Fractions					▣	▣		
Percentages					▣	▣		
Proportionality Concepts					▣	▣	▣	
Proportionality Problems					▣	▣	■	■
2-D Coordinate Geometry					□	□	■	■
Geometry: Transformations						▣	▣	▣
Negative Numbers, Integers & Their Properties						□	▣	
Number Theory							▣	□
Exponents, Roots & Radicals							▣	▣
Exponents & Orders of Magnitude							□	□
Measurement Estimation & Errors							□	
Constructions w/ Straightedge & Compass							■	□
3-D Geometry							▣	■
Congruence & Similarity								■
Rational Numbers & Their Properties								□
Patterns, Relations & Functions								□
Slope & Trigonometry								□
Number of topics covered by at least 67 % of the A+ countries	**3**	**3**	**7**	**15**	**20**	**17**	**16**	**18**
Number of Additional topics intended by A+ countries for typical curriculum	**2**	**6**	**5**	**1**	**1**	**3**	**6**	**3**
Total Topics typical A+ country	**5**	**9**	**12**	**16**	**21**	**20**	**22**	**21**

□Intended by 67% A+ countries; ▣Intended by 83% A+ Countries: ■Intended by 100% A+ Countries

Figure 2 – State Composite: Mathematics Topics Intended at Each Grade by At Least Two-thirds of 21 U.S. States

Topic	Grade 1	2	3	4	5	6	7	8
Whole Number Meaning	■	■	■	■	▣	□		
Whole Number Operations	■	■	■	■	▣	□		
Measurement Units	□	▣	▣	▣	■	▣	□	□
Common Fractions	□	■	■	▣				
Equations and Formulas	□	□	▣	▣	▣	■	■	■
Data Representation & Analysis (Graphing)	■	■	■	■	■	■	▣	
2-D Geometry: Basics	▣	▣	▣	▣	▣	▣	▣	▣
Polygons & Circles	■	■	■	■	■	■	■	▣
Perimeter, Area & Volume				▣	▣	▣	▣	□
Rounding & Significant Figures								
Estimating Computations	□	□	▣	▣	▣	▣	▣	▣
Properties of Whole Number Operations	□	□	□	□				
Estimating Quantity & Size			□					
Decimal Fractions			□	▣	▣	▣	□	□
Relationship of Common & Decimal Fractions				□	□	□		
Properties of Common & Decimal Fractions								
Percentages					□	▣	▣	□
Proportionality Concepts					▣	□		
Proportionality Problems						▣	▣	▣
2-D Coordinate Geometry			□	▣	□	□	□	▣
Geometry: Transformations	▣	▣	▣	▣	▣	▣	▣	▣
Negative Numbers, Integers & Their Properties						□	▣	□
Number Theory					▣	□	□	□
Exponents, Roots & Radicals						□	□	■
Exponents & Orders of Magnitude					□	□		
Measurement Estimation & Errors	□	□	▣	□	▣	▣	▣	□
Constructions w/ Straightedge & Compass								
3-D Geometry	■	■	■	▣	■	▣	■	▣
Congruence & Similarity					□	▣	▣	□
Rational Numbers & Their Properties						▣	▣	□
Patterns, Relations & Functions	▣	■	■	■	▣	■	■	■
Slope & Trigonometry								
Number of topics covered by at least 67 % of the states	**14**	**15**	**18**	**18**	**20**	**25**	**23**	**22**
Number of Additional topics intended by States for typical curriculum	**8**	**8**	**7**	**8**	**8**	**5**	**6**	**6**
Total Topics typical U.S. states	**22**	**23**	**25**	**26**	**28**	**30**	**29**	**28**

□Intended by 67% of states; ▣Intended by 83% of states: ■Intended by 100% of states

A cursory study of the three figures supports the TIMSS researcher's conclusions that the U.S. curriculum is not *focused* , since in comparison to the A+ countries there are far too many topics, particularly in the lower grades. Simple math can be used to conclude that each topic will be taught with much less depth in the U.S. The figures also support the researcher's contention that the U.S. curriculum is *highly repetitive*, since in comparison to the A+ countries, topics are introduced much earlier and are endlessly repeated. This characteristic is known as "spiraling" in U.S. educational circles, and some educators claim that as an advantage.

In regard the researcher's somewhat more sophisticated contentions that the U.S. state's curriculum is "not very demanding", especially in the middle school years, and is "incoherent" (not presented in logical, step-by-step order), it is recommended the reader study the *Definitions* section of the Appendix A, or better yet study the underlying TIMSS report, Section III, "Repetition and Incoherence in the U.S.", pages 10-16 [1]. Their contention that the U.S. curriculum is not presented to the students in a logical, step-by-step order is not hard to understand, however, since 22 topics start simultaneously in the 1st grade, as opposed to 5 topics in the A+ countries.

District Standards: The TIMSS researchers studied school district standards within a variety of U.S. states, and found these tended to include slightly fewer topics than are specified at the state level. Overall, they found the districts' standards were nearly as incoherent as the states' standards. They conclude that teachers are forced to cut back from what is intended, and that even the best teachers will have a difficult time trying to "distill a coherent curriculum from the incoherence that is offered them. Further, teachers are likely to prune back the state/local standards in different, idiosyncratic ways. This is what leads to the well-known American phenomenon – and special bane of transfer students – in which what's actually taught in a given grade varies wildly from class-to-class, even in the same school, district, or state".

In a related and sinister development, some U.S. school districts employ "math police" to be certain the local curriculum and teaching method is being exactly followed, and that teachers are not introducing extraneous material such as "algebra", as in one Ann Arbor, Michigan case. The huge New York City school district currently employs hundreds of math police thinly disguised as "coaches". Hiding the actual curriculum and textbooks from the math police is a sport engaged in by many high-performing public schools. All too often, teachers are threatened with firing or demotion.

Intended Content of California – The California Priority System: The state of California adopted new math standards in 1997, featuring a more focused mathematics curriculum, coherent from one year to the next, with a primary goal of having students fully ready for success in California Algebra I by the end of the seventh grade. One striking feature of this curriculum was the emphasis on pre-algebra starting in the first grade, and the introduction of symbolic algebraic thinking in the 4th grade. This curriculum was derived from a study of the most successful foreign countries in math performance, but with additional topics added as a result of the bitter political fight over the adoption of this radical (for the U.S) curriculum.

Starting in 2000, a series of decisions were implemented which effectively created a documented priority system in California, allowing parents, teachers and school districts easy access to a "core curriculum", as well as guidance regarding the relative importance of topics over and above the core curriculum. This was in the form of the Key Standards modification to the 1997 standards, the establishment of the California Standards Test (CST) to test all California students against the standards, and the High School Exit Exam (HSEE) to require algebra competence as a condition for graduation.

The "Key Standards" approach was implemented in 2000 [2], creating the basic component of the priority system and virtually guaranteeing that every child of even average math ability will be well prepared for 8th grade algebra. This was accomplished by greatly reducing the number of standards and allowing all students to focus on a core curriculum, including the strong pre-algebra component. The Key Standards comprise 30% of the total list of standards. The appendix contains a brief history of the tumultuous events surrounding the creation of the 1997 California standards and of the 2000 Framework document which implemented the Key Standards system.

A list of topics by grade has been created for California by the author using the same process as for the other states, as shown on Figure 4. This figure shows topics derived from Key Standards as black squares, and additional topics derived from non-key standards as open boxes. It will be noticed that topics derived from Key Standards make up approximately 80% of the total topics, even thought the Key Standards represent only 30% of the total number of standards. This is because each Key Standard is surrounded with a number of auxiliary standards which add more variations to the core subject, but do not appreciably add to the basic concept. An example is time telling in the 3rd grade, which is not a key "Measurement & Geometry" standard, but nevertheless falls under the topic of "Measurement units".

The California Standards Test (CST) was also implemented in 2000. The CST is the mechanism for testing California students on their understanding of subjects set forth in the California Standards, and is documented in the CST blueprint [6]. Each test has 65 multiple choice questions, with a minimum of 70% derived from Key Standards. The blueprint specifies how many questions will be asked on each of the four major strands ("Number Sense", "Algebra & Functions", "Measurement & Geometry", and "Statistics, Data Analysis and Probability"). The fifth strand, "Mathematical Reasoning", has no Key Standards, no multiple choice questions, is not included in the California approved textbooks, and is instead "embedded" into the other strands. This strand can be presumed to be a relic of the political fighting over the adoption of the standards.

The California High School Exit Exam (HSEE) is being implemented for the current school year, and is documented with it's own blueprint [7]. An excellent teacher's guide is also provided, including example questions [8]. This test is to be taken by each student after the completion of Algebra I, and generally occurs during the 10th grade. The test has 75 multiple choice questions, taken from 6th and 7th grade standards, and from Algebra I. Although this test is somewhat easier than one derived entirely from the Key Standards, it is still a very difficult test to pass for anyone who lacks a good understanding of basic algebra. Questions are taken from all four of the major strands, but at least 72% of the questions will be tough to answer without an algebraic background, which illustrates the close tie between algebra and the other major strands.

Figure 4 – California Composite: Mathematics Topics Intended at Each Grade based on the California Math Standards (Through Grade 7)

Topic / **Grade**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Whole Number Meaning	■	■	■	■	■		
Whole Number Operations	■	■	■	■	■	■	
Measurement Units	□	■	■	□	■	□	■
Common Fractions		■	■	■	■	■	
Equations and Formulas			■	■	■	■	■
Data Representation & Analysis (Graphing)	□	■	■	□	■	■	■
2-D Geometry: Basics	□	■	■	□	■	■	■
Polygons & Circles			■	□	■	■	
Perimeter, Area & Volume			■	□	■	■	□
Rounding & Significant Figures			□	■	□		
Estimating Computations	□	□	■	□			
Properties of Whole Number Operations		■		■	■	■	
Estimating Quantity & Size						□	
Decimal Fractions		■	■	■	■	■	
Relationship of Common & Decimal Fractions			□	■	■	■	
Properties of Common & Decimal Fractions				□	■	■	■
Percentages						■	■
Proportionality Concepts						■	■
Proportionality Problems						■	■
2-D Coordinate Geometry				■	■	□	■
Geometry: Transformations							
Negative Numbers, Integers & Their Properties				■	■	■	
Number Theory							
Exponents, Roots & Radicals					■		■
Exponents & Orders of Magnitude							
Measurement Estimation & Errors							
Constructions w/ Straightedge & Compass					■		□
3-D Geometry			□	□	■	□	■
Congruence & Similarity				□			■
Rational Numbers & Their Properties							■
Patterns, Relations & Functions	■	■	■	■			■
Slope & Trigonometry							■
Uncertainty & Probability					□	■	■
Real Numbers							■
Topics Intended – Key Standards	**3**	**9**	**12**	**11**	**18**	**16**	**17**
Additional Topics, Brighter Students	**2**	**1**	**2**	**3**	**2**	**3**	**2**
Additional Topics, All Non Key Standards	**4**	**1**	**3**	**9**	**2**	**4**	**2**

■ Key Standard Topic □ Non-Key Standard Topic

Considering the various features of the California priority system described above, one comes to the conclusion that California has at least two versions of an "intended topics" list, as follows:

1. Average Student: (Key Standards): Every student must learn this material in order to be ready for 8th grade Algebra I and for the algebra exit exam from high school. For the average student this is the effective math standard. The topics covered in this category are shown by the lower blue line on the graph of Figure 3. Note that any student who learns all this material will be guaranteed a 70% score on the CST, which is far above the current statewide average score of about 54%, and just about equal to that of a high performing school district such as Manhattan Beach. All the topics included in the HSEE are covered by 6th and 7th grade Key Standards topics (84% of questions) or by Algebra I, which is not included in the Key Standards system (16% of the questions). Any student with a good understanding of the Key Standards can answer all of the questions derived from the 6th and 7th grade portion of this test, and should be well prepared to learn the really basic Algebra I material included in this test.

2. Bright Student (More standards within each topic, and increased number of topics): There are some topics which are not covered by a Key Standard, yet are only taught in a few grades. For the brighter student, it makes sense to include these in a list of intended topics. This supplementary list of topics was compiled by noting that graphing and the various geometry subjects are taught in virtually every grade as Key Standards, and thus it is not necessary to add them to the Key Standards list in order to prepare a student for the highest level of achievement and for Algebra I. The total topics covered by this category are shown on the second to bottom line of Figure 4, and by the broken black line of Figure 3. The graph of this category is seen to be very close to that of the A+ countries. Although a political triumph of epic proportions, this result is not surprising considering the origin of the California curriculum.

Conclusions: The most notable feature of the data shown in Figure 3 is the dramatic reduction in topics in the critical first four grades (or alternately, the outrageous number of topics inflicted upon most of the poor kids of North America in those early years).

An example of the problem created by the U.S. curriculum relates to addition and subtraction of multi-digit numbers. The inability to learn how to add and subtract multi-digit numbers using carrying and borrowing or "regrouping" (California) or "renaming" (Singapore), or by any other method, is the first large barrier to success in math in the higher grades, according to personal interviews by the author with 3rd and 4th grade teachers. No matter which method is taught, the underlying concepts are critical to the understanding of multiplication and division, and of algebraic ideas. If a child gets as far as grade 4 without the ability to add and subtract multi-digit numbers, and without the related context knowledge, it is very difficult to catch up. In Singapore and in California, this material is taught in the 2nd grade, and reviewed in the 3rd grade. Those jurisdictions have the luxury of being able to concentrate heavily on this skill and on all the skills required previous to these lessons. In most North American jurisdictions this subject is introduced in the 3rd grade, but has to share time with 25 other math topics, and consequently many children without the help of sophisticated parents or tutoring fall behind and are still struggling in the 4th and 5th grade. This is an example of the advantages of the focused curriculum. In a companion paper [3], Bishop and Hook reach the following related conclusions about the performance results of the California experiment, based on 5 years of SAT-9 test data from 97% of the roughly 2.9 million California elementary school students:

> "Only in California have the TIMSS conclusions been put into practice. The list of Key Standards reduces the number of topics to a quantity roughly the same as that of the A+ countries (five topics in the 1st grade, ten in the 2nd grade, etc.). In regard to coherence, an examination of the Algebra and Functions strand, for instance, shows the careful and logical build-up of algebraic reasoning starting in the 3rd grade, with no topic nor problem introduced until the child has the tools to handle it. With the more focused, coherent sequence of topics, teachers can spend much more time on the important ones, and all teachers and parents will know exactly what will be covered by the state tests."

The TIMSS authors conclude their findings are even more important for the average student or economically disadvantaged student than for the college bound one. The Schmidt paper contains an excellent sidebar discussing this issue [9]. It makes specific reference to the widening gap between American students and Asian students as they progress to the higher grades, as shown in the 1995 TIMSS and on Figure 5 of this paper,

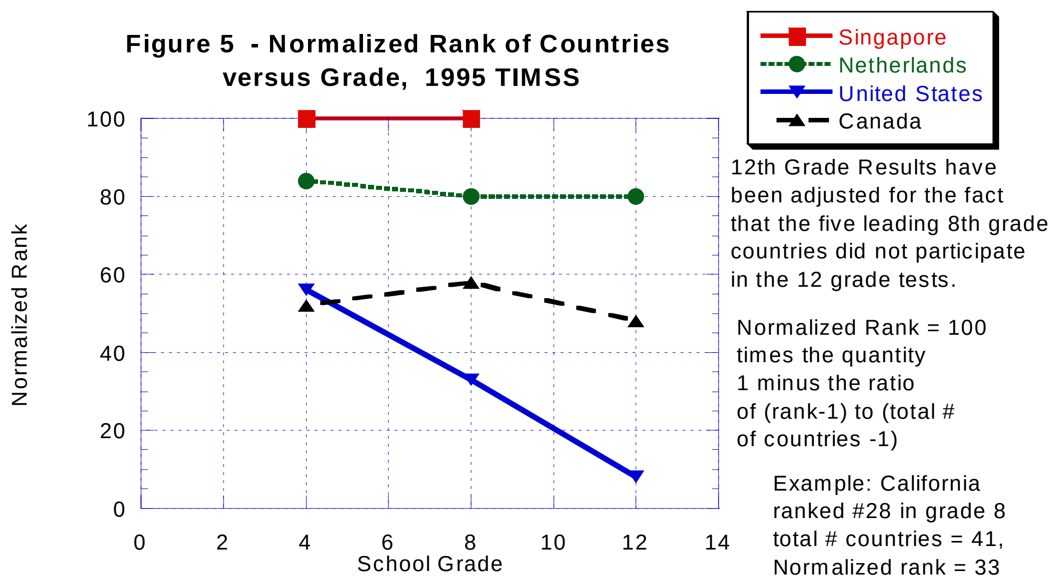

Figure 5. 1995 TIMSS ranking results for all three grades tested. The ranking results have been normalized to account for the different total number of countries participating in each grade. Since the five leading countries in the eighth grade test did not participate in the 12 grade tests ("final year of secondary school"), those rankings were adjusted to assume those five countries also would have come first in the 12 grade tests. This data is the origin of the statement "the longer a student stays in a U.S. public school, the further he or she falls behind". The 1999 TIMSS showed the same downward trend.

and note an eerie similarity to the widening gap inside American schools between advantaged and disadvantaged students as they progress through the grades. The reasons advanced relate to the unfair advantage possessed by the student with well-off or sophisticated parents. In particular, they refer to the student who is lucky enough to have gained the needed background knowledge at home or via tutoring versus those who must depend on what they get sporadically from the schools. The learning of the luckier students snowballs while that of the less fortunate ones – those dependent on the incoherent American curriculum - never begins to gather momentum.

In the A+ countries and in California, all the students are the fortunate ones.

References

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9. E.D. Hirsch, Jr., Professor Emeritus of Education and Humanities at the University of Virginia. Sidebar found on pages 6 and 7 of reference #1, and taken from "The Schools We Need and Why We Don't Have Them", 1995, Doubleday, by professor Hirsch.

Appendix

I. Background and Details, TIMSS 1995 Curriculum Study Conclusions

What is the source for this material?:

William Schmidt, Richard Houange, and Lelend Cogan, "A Coherent Curriculum, The Case of Mathematics", published in American Educator, Summer 2002 [1]

William Schmidt is the director of the U.S. National Research Center for the Third International Math and Science Study (TIMSS) and University Distinguished Professor at Michigan State University. Richard Houange is the associate director, and Leland Cogan is a senior researcher.

Major conclusions: The curriculum itself - what is taught – makes a huge difference in the performance of the various countries. The U.S. curriculum has way too many topics intended to be taught, particularly in the early grades; the intended content is highly repetitive, incoherent and not very demanding in the middle-school years.

What is actually taught
Researchers examined the "Intended Content" (National Curriculum or State Standards) in 37 countries. They found the "intended content" is essentially replicated in the nation's Textbooks. They also found in most countries studied, teachers "follow" the textbook, guided by the depth and duration of each topic in the textbook

> Schmidt et. al.'s conclusion: "It can be said with statistical confidence that what is stated in the "intended content", and in the textbooks is, by and large, taught in the classrooms of most TIMSS countries."

Top achieving (A+) countries:

The 42 participating countries were ranked using their 8th grade (mean) score. The six highest ranking countries were selected, each of which statistically outperformed at least 35 other countries. These countries were designated the A+ countries, and were Singapore, Korea, Japan, Hong Kong, Belgium and the Czech Republic

Common Topics

An extensive list of math topics was given to Education officials and researchers in each of the A+ countries, and the topics intended for each country, by grade, were compiled, based on the national curriculum (intended content). A list of these topics is included at the end of this appendix. Figure 1 shows the topics which were intended by at least 2/3 of the A+ countries, as well as the average number of additional intended topics not meeting the 2/3 criteria.

Figure 2 shows the same information for the U.S., indicating the intended topics for at least 2/3 of the 21 States participating, plus the average number of additional intended topics not meeting the 2/3 criteria. All the topics to the left of the red dotted line are not taught in the A+ countries at the indicated grade. It can be seen there are far more topics taught in the U.S, particularly in the early grades.

Figure 3 shows the total number of topics taught in the A+ countries by grade, compared to the total number of topics taught in the U.S. by grade. It can be seen that the number of topics taught in the U.S. in the critical early grades ranges from over four times as many in grade 1 to over twice as many in grade 3.

II. The Case of California: It was <u>not</u> one of the states participating in the 1995 TIMSS study.

In the 1990's California underwent a bitter and hard fought transition from the old NCTM-based standards to the present Key Standard system.

In 1994 and 1995, groups of California parents became concerned with the content of their children's school math programs [4]. Among them were educators and professionals in many areas. University professors in biology, economics, mathematics, the sciences and statistics joined with the parents. After rebuffs at the local level, they approached the governor (Republican Pete Wilson) and the legislature, and found a concern there as well. In response to those concerns, legislation was passed in 1996 requiring new state standards and frameworks in all major content areas. The legislation mandated standards at the level of the top achieving countries in the world, in order to prepare California's children to compete in the global economy.

The first math commission to carry out this mandate had no professional mathematicians, and produced a document filled with errors and lack of focus. This version was rejected, and mathematicians from Stanford University were hired to remove mathematical errors, to rearrange the order of standards so the basic skills appear earlier than the skills which depend upon them, and to remove all recommendations regarding teaching methods. A second version was published. After much debate, public hearings, expert testimony, all with heavy press coverage, a compromise version of the Stanford math standard document was adopted in 1997.

Although this version was generally good (rated #1 out of 48 by the Fordham Foundation), it still had far too many subjects to qualify as "equivalent to the top achieving countries in the world". This was a result of intense lobbying by the math education industry, comprising the education faculties, math teachers associations and the textbook industry . It was a horrendous fight every step of the way, widely covered by the press, and the reformers were forced to compromise in order to get the new standards accepted [5].

This final problem was solved in 2000, with the adoption of the California Mathematics Framework document [2], written by the same Stanford mathematics professors who were hired to fix the original standards document. Their framework document identified a set of Key Standards, which are now in fact "the" basic California Standards. This (after the fact) Key Standards system was the way to compensate for the earlier compromises that had to be made to get there at all. One entire strand (Mathematical Reasoning) has no Key Standards at all, is not included in at least one California approved textbook series (Harcourt Math), and has been effectively eliminated from the

California curriculum.   The number of grade 1-7 Key standards is exactly 30 % of the total math standards in the 1997 document.

The topics contained within the California Key Standards are shown in Figure 4 for grades 1-6, and the total number of topics for those grades are included in Figure 3 for both the Key Standards to represent the average student, and a larger set of standards to represent the bright student..   It appears certain, when considering Figures 1, 3 and 4, that the California Key Standards do qualify to be rated "equivalent to the top achieving countries in the world"

Virtually all the other North American states and provinces can trace their math standards back to the 1989 NCTM Standards or to the related math reform movements of that era. Although the quantity of subjects shown as the top plot of Figure 3 is a composite of the topics from the 21 states that volunteered to participate in the 1995 TIMSS, it is safe to say that this data applies to virtually all the U.S. states and Canadian provinces.

III. Definitions: The words "coherent" and "focused" are used in a very special and unique manner in the Schmidt paper, and in math curriculum writings from other authors. In order to avoid confusion, a summary of the definitions used in Schmidt et. al. is given below:

<u>Coherent</u> (Schmidt et. al., page 9): "We define content standards and curricula to be coherent if they are articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter flows." Further, the content standards must evolve from particulars to deeper structures inherent in the discipline. The authors give as an example "the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fraction" which evolves into "an understanding of the rational number system and it's properties". These are mighty big words to use on parents. If anyone asks what we mean by "coherent", this author is inclined to assert two properties: (1) subjects should be presented in a logical order, so one builds on another and (2) the student should become increasingly aware of the generality of the material he or she is learning. An obvious example is how the simple number exercises of demonstrating that 3 + 5 = 5 + 3 in the 2^{rd} grade evolves into a general commutative law of algebra by the 8^{th} grade, as presented in the California Framework document.

> *Simple Definition*: Coherent = subjects presented in logical order, and proceed from the specific to the general.

<u>Focus</u> (Schmidt et. al., page 3): "Our (*the U.S.)* intended content is not focused. If you look at state standards, you'll find more topics at each grade level than in any other nation . . . . eighth grade math textbooks in Japan have around 10 topics, but U.S. eighth grade textbooks have over 30 topics."

> *Simple Definition:* A focused curriculum = one with a reduced number of topics taught, so that students can focus on the core topics

IV. Mathematics Topics: http://currmap.ncrel.org/mathTopicsList.htm

Numbers

- Whole Numbers
  - Whole Numbers: Meaning
  - Whole Numbers: Operations
  - Whole Numbers: Properties of operations
- Fraction and Decimals
  - Common fractions
  - Decimal fractions
  - Relationships of common and decimal fractions
  - Percentages
  - Properties of common and decimal fractions
- Integer, Rational and Real Numbers
  - Negative numbers, integers, and their properties
  - Rational numbers and their properties
  - Real numbers, their subsets, and their properties
- Other Numbers and Number Concepts
  - Binary arithmetic and/or other number bases
  - Exponents, roots, and radicals
  - Complex numbers and their properties
  - Number theory
  - Counting
- Estimation and Number Sense
  - Estimating quantity and size
  - Rounding and significant figures
  - Estimating computations
  - Exponents and orders of magnitude

Measurement

- Measurement and Units
- Perimeter, area, and volume
- Estimation and errors

Geometry: Position, Visualization, and Shape

- Two-dimensional geometry: coordinate geometry
- Two-dimensional geometry: basic
- Two-dimensional geometry: polygons and circles
- Three-dimensional geometry
- Vectors

Geometry: Symmetry, Congruence, and Similarity

- Transformations
- Congruence and similarity
- Constructions using straight-edge and compass

Proportionality

- Proportionality concepts
- Proportionality problems
- Slope and trigonometry
- Linear interpolation and extrapolation

Functions, Relations, and Equations

- Patterns, relations, and functions
- Equations and formulas

Data Representation, Probability, and Statistics

- Data representation and analysis
- Uncertainty and probability

Elementary Analysis

- Infinite processes
- Change (growth and decay, differentiation)

Validation and Structure

- Validation and justification
- Structuring and abstracting
- Other content

1