High School Math Programs that Work

Comments of Fred Greenleaf, Professor of Mathematics, New York University, presented at a panel organized by the Community Education Council of District 2 (New York City), June 1 2005.

I speak tonight as a member of NYCHOLD, an association of parents, teachers, mathematicians, and scientists committed to improving K-12 math education in New York City; I also speak for my many colleagues in the professional mathematics community who share these concerns. My remarks will be directed to the question

> *What is needed in a high school math curriculum if its graduates are to succeed in college-level programs with significant math content?*

I have also been asked to comment on the ARISE math curriculum currently mandated for most high schools in District 2. My comments on that will be brief: as I will explain, ARISE is totally inadequate for the job.

There are some college programs where math preparedness might not be a big issue, but it will be crucial if your child ends up wanting to pursue interests in

Architecture	Biosicence	Engineering
Business	Computer Science	Environmental Science
Medicine	Economics	Physical Sciences

and of course, mathematics and secondary math education. Being "calculus-ready" upon entry to college is no longer an issue affecting only those who aspire to become astrophysicists; it now affects all ethnic and economic groups. Recent studies reveal some striking facts[1]

- *Nearly 75% of U.S. high school graduates enroll in college* (*including junior colleges*) *within two years of graduation*

But many are ill-prepared to succeed. Even among those taking a core college prep high school curriculum – four or more years of English and three years each of math, social sciences, and natural science – many are still not prepared to succeed in credit-bearing first year college courses.

- *Not only is taking the* RIGHT NUMBER *of courses important, but taking the* RIGHT KIND *of courses is critical to student readiness for college level work.*

Students aspiring to these many programs will find themselves out of luck unless their high school has a sound college-oriented math curriculum. A bad math curriculum is particularly hard on those whose parents cannot afford the extensive tutoring needed to make up for the deficiencies of their high school (and indeed K-12) math programs.

[1] *On Course for Success: A Close Look at Selected High School Courses that Prepare All Students for College*, a 2004 report by The Education Trust foundation and the nonprofit ACT educational testing service with which it is affiliated. This document can be found on the www.nychold.com website, or accessed at the ACT website: www.act.org/path/policy/pdf/success-report.pdf

What's Wrong with our Math Curricula?

The reason I'm here tonight is that I know what it takes to succeed at college level math. And, from years of work developing entry-level college math courses, I am acutely aware that the greatest single cause of failure in these courses is lack of fluency in

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By this I mean the practiced ability to deal with equations, set them up from verbal descriptions, and correctly solve them using the tools that algebra provides. Constructivist math programs at all K-12 levels have greatly downplayed the importance of symbolic reasoning and algebra, often in favor of mindless manipulation of numerical data on a graphing calculator. In college-level math the *last* thing you do is reach for the calculator, after you have thoroughly analyzed the problem, set up a mathematical model, and solved it using algebraic methods.

As a preparation in algebra the ARISE curriculum is a total failure, from beginning to end. I reviewed these texts 6 years ago when I first began to investigate K-12 math curricula being introduced in District 2, and have reviewed them again in the past month. Aside from the fact that the chapters appear to have been written by committees that were not speaking to each other, I cannot begin in this brief talk to lay out all the deficiencies of this program. Instead, let me cite a just a few facts about the ninth grade text ARISE 1, in which students should be intensively working on algebra skills. In the entire book we find

- Total number of text pages = 691
- Number of pages *on which there appears any sort of discussion applying algebra to an actual equation to solve a problem* = 52 (about 7%).
- Largest number of consecutive pages *without a single equation appearing* (whether or not used to find a solution) = 98
- *Next longest* stretch of consecutive pages *without a single equation appearing* (whether or not used to find a solution) = 74

And so it goes. I rest my case regarding ARISE.[2]

For the present audience it is hardly necessary to explain what is meant by "constructivist math programs." When it comes to mathematics and the sciences, the philosophical basis for constructivist programs such as TERC (in grades K-5) and ARISE (in grades 9-12) is simply absurd. Consider two of its central tenets

- The only valid way for students to achieve deep understanding of mathematics is to discover things for themselves, working in small inquiry groups of their peers.

[2]Incidentally, this is not the worst of the constructivist high school math curricula. That honor goes to the IMP program (Interactive Math), which was once proposed for wide use in the Bronx superintendancy, including Bronx Science. The first book in that series (ninth grade) is 571 pages long. It contains a grand total of 23 pages on which the symbol "=" (equal sign) appears! This curriculum is still being used by one school in the Bronx, but the attempt to impose it on wide scale died in the face of widespread teacher protest.

- The role of the teacher is not to instruct students directly. Teachers should be the "guides on the side," as students discover things on their own and "construct their own knowledge."

How *did* anyone in this audience, educated in those dark benighted times before constructivist math programs arrived on the scene, ever manage to get through college and become successful professionals?

Since prehistoric times, human beings have survived and prospered by being able to pass on the accumulated knowledge of our species to our children. In the present era we have vast bodies of information to deal with; some areas (medicine, mathematics) have taken hundreds or even thousands of years of patient experiment, observation, and false starts to evolve into their present forms. Passing all this knowledge from one generation to the next requires efficient and effective methods, namely flexible and innovative *direct instruction* by *teachers* who are highly competent in their subjects. Having students sit around re-inventing the wheel in endless trial-and-error "discovery projects" is not an option.

What Works?

"Direct instruction" means having an experienced and knowledgeable teacher who speaks directly to students, sets the agenda, presents facts, reminds everyone of relevant background concepts, and leads students in coming to grips with the subject of the day. Despite efforts of the constructivists to demonize this mode of instruction as being based on "rote learning," it has time and again proved to be the single most effective mode of instruction for all students.

During the period 1967-1976 (with follow up studies extending through 1996) *Project Follow Through*,[3] a nationwide study involving 70,000 students in 180 schools, compared the results of direct instruction in math and reading with various constructivist approaches: discovery learning, whole language, etc. Only one approach, the *direct instruction* model, consistently produced students who were near or at national norms in math and language, and close to national norms in reading. Students taught in the other modes did worse, often worse than the control group. This study demonstrating the superiority of direct instruction methods (for all income and racial groups) has been systematically ignored by proponents of the constructivist curricula now being foisted on our schools.

More recently, in 2004 the Education Trust foundation and the nonprofit ACT educational testing service with which it is affiliated issued a report on high school programs, including mathematics, that work in preparing students for college.[4] The premise of this study was simple. Investigators first identified a nationwide sample of high schools with the following characteristics.

[3] A recent account of this study, its implications, and the response of the constructivist education community can be found in an article sponsored by the Fordham Foundation *Why Education Experts Resist Effective Practices* (*And What It Would Take to Make Education More Like Medicine*), by Douglas Carnine. It can be accessed on the www.nychold.com website, or at: www.edexcellence.net/library/carnine.html.

[4] *On Course for Success: A Close Look at Selected High School Courses that Prepare All Students for College*, a 2004 report by The Education Trust foundation and the ACT educational testing service. This document can be found on the www.nychold.com website, or accessed at the ACT website: www.act.org/path/policy/pdf/success-report.pdf

- A high percentage of minority and low-income students (at least 40% minority and/or at least 50% low-income).

coupled with a substantial number of good outcomes among their college-bound students on a battery of ACT Assessment tests:[5]

- At least 65% of all students had good outcomes on the ACT English Assessment Test

AND

- At least 35% of all students had good outcomes on the ACT Math Assessment Test and/or at least 24% had good outcomes on the ACT Science Assessment Test.

In the mathematics study investigators identified all students who met the achievement criteria in each of these schools and asked them

> *Which high school math classes had they taken and who were their teachers?*

Finally, the study spent two years at each school evaluating the classes offered and the instructional methods that produced these results, with particular attention to the basic core of college preparatory math courses: Algebra I, Algebra II, Geometry, and Trigonometry.

These successful math programs were all found to have certain common characteristics:

- Qualified and experienced teachers
- Strongly college-oriented course content
- Teaching that was flexible and responsive to students
- Extra student tutorial support when neeeded

In particular, all teachers in the study were certified in their subject, with at least a bachelor's degree in that content area (not an Ed School degree). Course content was aimed toward successful transition to college, and was often at a level beyond that of state and district standards.[6]

The predominant mode of instruction was "exposition and questioning" – with a teacher actively explaining concepts and asking questions to check for student understanding. Though pedagogy was teacher-directed (*direct instruction*, if you will), there was a constant flow of questioning to and from the audience. Each lesson had a definite topic, and began with a review of background concepts as necessary ; lessons, assignments, and projects were focused on one topic at a time. Careful attention was paid to definitions, correct use of mathematical language

[5] In mathematics, for instance, a "good outcome" was defined to be an ACT score that predicts a 75% chance of of getting a C or better on the student's first credit-bearing college math course. A separate study by the U.S. Department of Education found a similar result when ACT scores were compared with likelihood of college graduation. Performance on that first math course was highly correlated with eventual college success.

[6] Keep in mind that AP programs were excluded from this study.

(as it will be encountered in college courses), and firm mastery of concepts in a natural order. Appropriate, but sparing, use was made of technology – graphing calculators, geometry programs, etc. These teachers all conveyed an obvious love for their discipline, and the conviction that all students could master their subject. None used what we would recognize as a constructivist approach or texts.

What is to be Done?

At the very least I would recommend the District 2 math specialists begin to pay attention to these studies, which demonstrate the superiority of intelligently formulated modes of direct instruction. They should, for example, take a close look at the ACT study; they might even get in touch with some of the schools involved – they are all identified – and ask what books and curricular models *they* used.

This critique applies to all grade levels. The trouble begins with the constructivist K-5 math curriculm (TERC) favored by District 2, which systematically avoids symbolic skills and thinking in favor of "manipulatives," seldom provides clear definitions of any concept, makes a confusing hash of the basic algorithms of arithmetic, and in all fails to equip students with mastery of such basic skills as knowing their multiplication tables, or the ability to handle fractions *as fractions, not as decimals punched up on a calculator, or as the measured lengths of strips of paper.*[7]

This aversion to basics undermines the foundation for later understanding of algebra, which is crucial to eventual success in college-level work. If the foundation is rotten, all later efforts are compromised.

[7]Constructivist math educators often attempt to justify these curricular choices by characterizing these topics as "rote learning." They are not; they are the essential foundation for later work in algebra, when symbols replace actual numbers. Those who insist on characterizing anything other than constructivist math curricula as "rote learning" should take a look at the article *In Defense of "Mindless Rote"*, by Ethan Akin, Professor of Mathematics at CCNY. It can be found on the NYCHOLD website: www.nychold.com/akin-rote01.html It is an informative read for parents and constructivists alike.