

A World Class K-7 Math Curriculum Verified by Outstanding Experimental Research

William Hook - 4 April 2004 (CA DOE web sites updated May 17)

Summary

Described here are two remarkable experiments, which taken together provide compelling evidence that British Columbia can have a world class grade K-7 math curriculum based on outstanding experimental research.

The first experiment involved over 500,000 worldwide students tested over a 4 year period, and the second involved over 2.9 million California elementary school students tested yearly over a 5 year period.

The first experiment involved math curriculum research, and is derived from data gathered during the Trends in International Math & Science Studies (TIMSS).

The TIMSS researchers concluded it is the curriculum itself - what is taught - which makes a huge difference in successful math learning.

- The researchers devised a method to (quantitatively) measure the characteristics of such a successful curriculum.
- No U.S. public school jurisdiction was found to have a successful math learning curriculum.

The second experiment involved synthesizing a new California curriculum based on the successful curriculum of the leading TIMSS countries, and testing it to verify this type of curriculum will work with North American students.

- Results were outstanding with average and with economically disadvantaged students, as well as with the bright students.
- Data is presented to support these conclusions.

Reasonable Goals for British Columbia

A world class mathematics curriculum verified by the best available experimental research.

A curriculum open and clear to parents, with specific written standards for each grade.

A curriculum which will properly prepare the average and the economically disadvantaged student for 8th grade Algebra I, where this student has neither a sophisticated parent nor funds for tutoring.

A curriculum to prepare all students for the best possible success in high school math consistent with their math aptitude, and for admission to post-secondary education (70% of new jobs).

A curriculum with matching textbooks for each grade, provided to each student.

Homework focused on the core subjects, and less of it.

The First Experiment - Discovering the Best Curriculum

Two very large math education research experiments were carried out in 1995 through 2002.

The first involved over 500,000 students tested, 45 countries, 15,000 schools. These were the TIMSS studies.

This was a conventional “discovery” experiment, where one finds out which countries are best in math, and attempts to discover what there is about those countries which makes them best [1].

The six leading (A+) math performance countries were identified by massive testing in 1995 and verified in 1999 (Singapore, Korea, Japan, Hong Kong, Belgium, Czech Republic):

- Rankings based on the average scores for all the 8th grade students

A variety of characteristics of participating countries and U.S. states were recorded during the 1995 tests.

One of these characteristics was curriculum. A method to quantitatively describe curriculum was devised and applied to the A+ countries, and to participating U.S. states:

- An extensive list of math topics was created by mathematicians (appendix).
- List given to education officials and researchers in each of the A+ countries.
- Topics compiled by grade based on the national curriculum (intended content).
- Figure 1 shows the topics which were intended by at least 2/3 of the A+ countries.
- Also shows average number of additional intended topics not meeting the 2/3 criteria.
- Figure 2 shows the same information for the participating U.S. states. All the A+ country's topics are to the right of the red line.
- Figure 3 shows a graph of the total topics intended by grade.

The U.S. curriculum was found to be, in comparison to the A+ countries:

- Not focused (far too many topics, particularly in the lower grades)
- Highly repetitive (topics introduced too early, too little depth, endlessly repeated)
- Incoherent (not presented in logical, step-by-step order)
- Not very demanding (especially in middle school years)

Curriculum was the only factor found to differ significantly between the A+ countries and the poor performing U.S. states. It was not teachers, not demographics, nor any other non-school factor.

The TIMSS researchers also found an ever-widening test score gap, as a function of grade level, between the U.S. and the foreign countries, as shown in Figure 4 [2].

Figure 1 – A+ Composite: Mathematics Topics Intended at Each Grade by At Least Two-thirds of A+ Countries

Topic	Grade	1	2	3	4	5	6	7	8
Whole Number Meaning		■	■	■	☐	☐			
Whole Number Operations		■	■	■	■	☐		☐	
Measurement Units		☐	■	■	■	■	☐	☐	
Common Fractions				☐	■	■	☐		
Equations and Formulas				☐	☐	☐	☐	■	■
Data Representation & Analysis (Graphing)				☐	☐	☐	☐		☐
2-D Geometry: Basics				☐	☐	☐	☐	■	■
Polygons & Circles					☐	☐	☐	■	■
Perimeter, Area & Volume					☐	☐	☐	☐	☐
Rounding & Significant Figures					☐	☐			
Estimating Computations					☐	☐	☐		
Properties of Whole Number Operations					☐	☐			
Estimating Quantity & Size					☐	☐			
Decimal Fractions					☐	■	☐		
Relationship of Common & Decimal Fractions					☐	■	☐		
Properties of Common & Decimal Fractions						☐	☐		
Percentages						☐	☐		
Proportionality Concepts						☐	☐	☐	
Proportionality Problems						☐	☐	■	■
2-D Coordinate Geometry						☐	☐	■	■
Geometry: Transformations							☐	☐	☐
Negative Numbers, Integers & Their Properties							☐	☐	
Number Theory								☐	☐
Exponents, Roots & Radicals								☐	☐
Exponents & Orders of Magnitude								☐	☐
Measurement Estimation & Errors								☐	
Constructions w/ Straightedge & Compass								■	☐
3-D Geometry								☐	■
Congruence & Similarity									■
Rational Numbers & Their Properties									☐
Patterns, Relations & Functions									☐
Slope & Trigonometry									☐
Number of topics covered by at least 67 % of the A+ countries		3	3	7	15	20	17	16	18
Number of Additional topics intended by A+ countries for typical curriculum		2	6	5	1	1	3	6	3
Total Topics typical A+ country		5	9	12	16	21	20	22	21

☐ Intended by 67% A+ countries; ☐ Intended by 83% A+ Countries; ■ Intended by 100% A+ Countries

Figure 2 – State Composite: Mathematics Topics Intended at Each Grade by At Least Two-thirds of 21 U.S. States

Topic	Grade	1	2	3	4	5	6	7	8
Whole Number Meaning		■	■	■	■	☐	☐		
Whole Number Operations		■	■	■	■	☐	☐		
Measurement Units		☐	☐	☐	☐	■	☐	☐	☐
Common Fractions		☐	■ I	■	☐				
Equations and Formulas		☐	☐ I	☐	☐	☐	■	■	■
Data Representation & Analysis (Graphing)		■	■ I	■	■	■	■	☐	
2-D Geometry: Basics		☐	☐ I	☐	☐	☐	☐	☐	☐
Polygons & Circles		■	■	■ I	■	■	■	■	☐
Perimeter, Area & Volume					☐ I	☐	☐	☐	☐
Rounding & Significant Figures									
Estimating Computations		☐	☐	☐ I	☐	☐	☐	☐	☐
Properties of Whole Number Operations		☐	☐	☐ I	☐				
Estimating Quantity & Size				☐ I					
Decimal Fractions				☐ I	☐	☐	☐	☐	☐
Relationship of Common & Decimal Fractions					☐ I	☐	☐		
Properties of Common & Decimal Fractions									
Percentages						☐ I	☐	☐	☐
Proportionality Concepts						☐ I	☐		
Proportionality Problems							☐ I	☐	☐
2-D Coordinate Geometry				☐ I	☐	☐ I	☐	☐	☐
Geometry: Transformations		☐	☐	☐	☐	☐ I	☐	☐	☐
Negative Numbers, Integers & Their Properties							☐ I	☐	☐
Number Theory						☐ I	☐	☐	☐
Exponents, Roots & Radicals							☐ I	☐	■
Exponents & Orders of Magnitude						☐ I	☐	☐	
Measurement Estimation & Errors		☐	☐	☐ I	☐	☐	☐ I	☐	☐
Constructions w/ Straightedge & Compass									
3-D Geometry		■	■	■	☐	■	☐ I	■	☐
Congruence & Similarity						☐ I	☐	☐ I	☐
Rational Numbers & Their Properties							☐ I	☐ I	☐
Patterns, Relations & Functions		☐	■	■	■	☐	■	■ I	■
Slope & Trigonometry									
Number of topics covered by at least 67 % of the states		14	15	18	18	20	25	23	22
Number of Additional topics intended by States for typical curriculum		8	8	7	8	8	5	6	6
Total Topics typical U.S. states		22	23	25	26	28	30	29	28

☐ Intended by 67% of states; ☐ Intended by 83% of states; ■ Intended by 100% of states

Figure 3 - Total Number of Topics Intended in the A+ Countries, in the U.S. (not including California), and in California

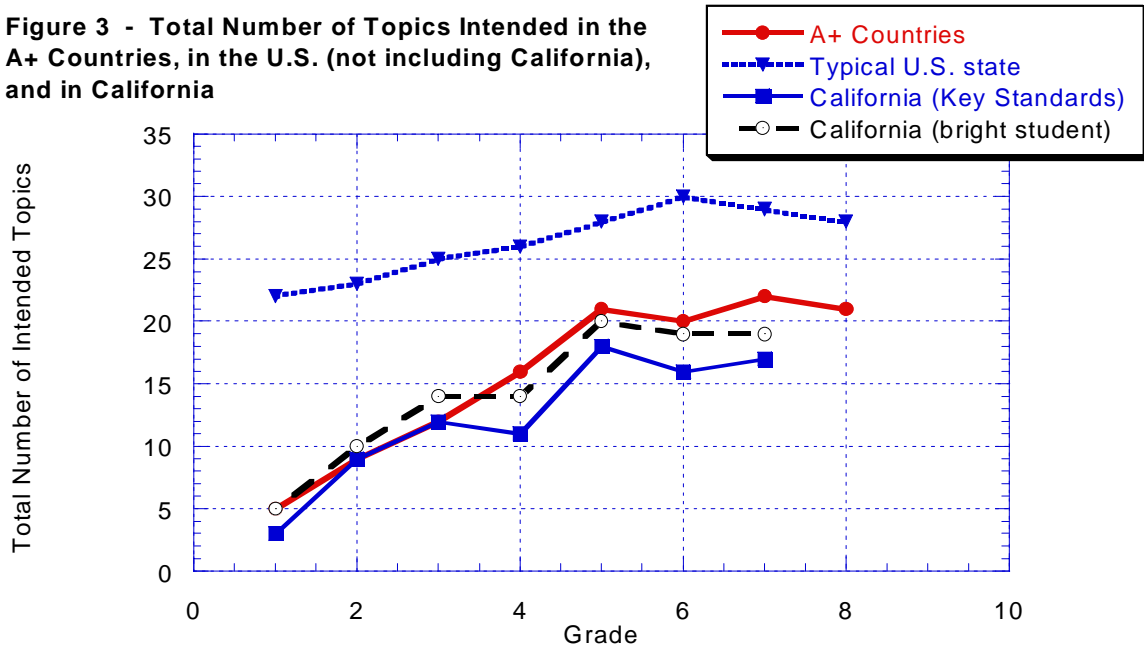


Figure 3. Showing the total number of intended topics from the TIMSS curriculum study for the A+ countries, the typical U.S. state, and for California. The graph for the California Key Standards is aimed at the average student, and is the core curriculum set forth in the California Framework Document. It accounts for a minimum of 70% of the questions on the yearly California Standards Test (CST). The graph for the bright student is based on a number of additional non-Key standards, as shown on the bottom of Figure 5.

Figure 4 - Normalized Rank of Countries versus Grade, 1995 TIMSS

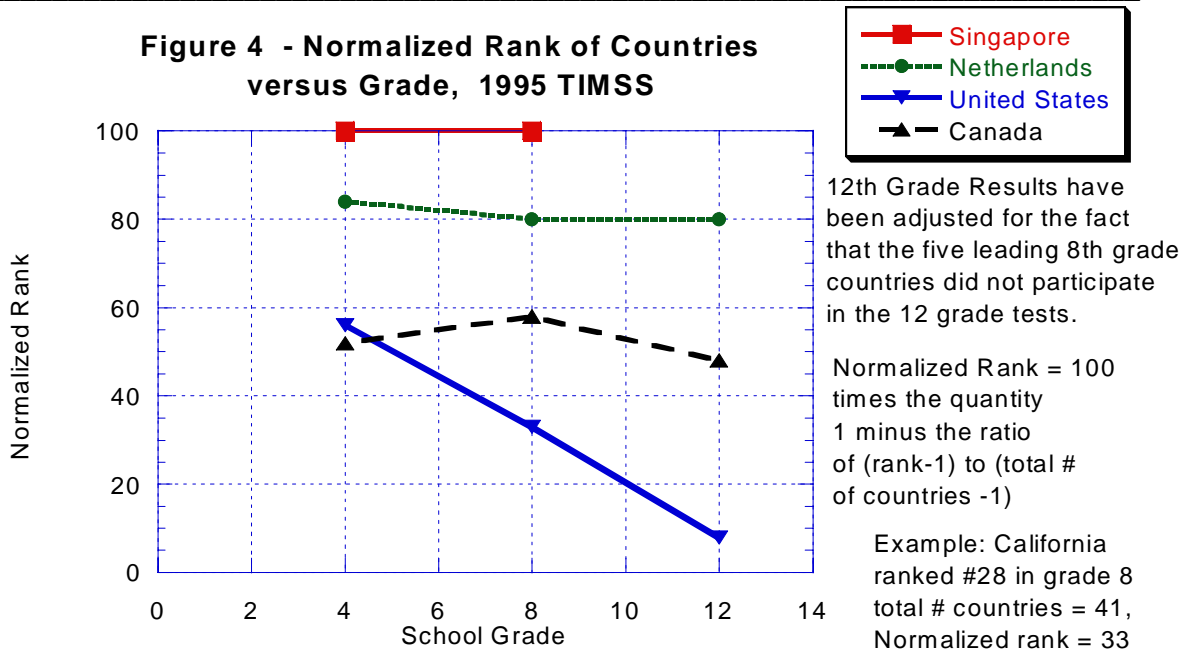


Figure 4. 1995 TIMSS ranking results for all three grades tested. The ranking results have been normalized to account for the different total number of countries participating in each grade. Since the five leading countries in the eighth grade test did not participate in the 12 grade tests (“final year of secondary school”), those rankings were adjusted to assume those five countries also would have come first in the 12 grade tests. This data is the origin of the statement “the longer a student stays in a U.S. public school, the further he or she falls behind”. The 1999 TIMSS showed the same downward trend.

They further noted the eerie similarity to the ever-widening gap between the children of well-off or sophisticated parents and those of the disadvantaged at the higher grades within the U.S.

- They concluded the U.S. curriculum favored the children of well-off or sophisticated parents who could provide supplementary tutoring, and was terribly unfair to the disadvantaged.
- “The learning of the luckier students snowballs while that of the less fortunate ones - those dependent on the incoherent American curriculum - never begins to gather momentum”.

The TIMSS researchers also studied School District Standards, and found slightly fewer subjects, but the same level of incoherence. They conclude teachers are forced to cut back from what is intended, and will have a difficult time trying to distill a coherent curriculum from the incoherence offered them. This leads to a wild variation from class to class, even in the same school or district.

The TIMSS research clearly shows it is the early grades where the damage is done, and it is the early grades which must be fixed.

The Second Experiment, Reversing the Process

This experiment involved over 2.9 million California elementary school students tested, and about 7,500 schools.

A California curriculum was synthesized from the essential characteristics of the A+ countries, and was adopted in late 1997, and modified into its final form in 2000, as shown in Figures 5 & 3 [3]:

- Key Standards priority system – Focussed, coherent, demanding in the middle grades, not overly repetitive (black squares in Figure 5).
- Every student prepared for 8th grade Algebra I.
- Features pre-algebra starting in 2nd grade, and the introduction of symbolic algebraic thinking in 4th grade.
- Optional topics for bright students (white squares in Figure 5).
- Math and English declared highest priority subjects in state.
- Yearly testing, high school algebra Exit Exam required to graduate.

Figure 5 - California Composite: Mathematics Topics Intended at Each Grade based on the California Math Standards (Through Grade 7)

Topic	Grade	1	2	3	4	5	6	7
Whole Number Meaning		■	■	■	■	■		
Whole Number Operations		■	■	■	■	■	■	
Measurement Units		□	■	■	□	■	□	■
Common Fractions			■	■	■	■	■	
Equations and Formulas				■	■	■	■	■
Data Representation & Analysis (Graphing)		□	■	■	□	■	■	■
2-D Geometry: Basics		□	■	■	□	■	■	■
Polygons & Circles				■	□	■	■	
Perimeter, Area & Volume				■	□	■	■	□
Rounding & Significant Figures				□	■	□		
Estimating Computations		□	□	■	□			
Properties of Whole Number Operations			■		■	■	■	
Estimating Quantity & Size							□	
Decimal Fractions			■	■	■	■	■	
Relationship of Common & Decimal Fractions				□	■	■	■	
Properties of Common & Decimal Fractions					□	■	■	■
Percentages							■	■
Proportionality Concepts							■	■
Proportionality Problems							■	■
2-D Coordinate Geometry					■	■	□	■
Geometry: Transformations								
Negative Numbers, Integers & Their Properties					■	■	■	
Number Theory								
Exponents, Roots & Radicals						■		■
Exponents & Orders of Magnitude								
Measurement Estimation & Errors								
Constructions w/ Straightedge & Compass						■		□
3-D Geometry				□	□	■	□	■
Congruence & Similarity					□			■
Rational Numbers & Their Properties								■
Patterns, Relations & Functions		■	■	■	■			■
Slope & Trigonometry								■
Uncertainty & Probability						□	■	■
Real Numbers								■
Topics Intended – Key Standards		3	9	12	11	18	16	17
Additional Topics, Brighter Students		2	1	2	3	2	3	2
Additional Topics, All Non Key Standards		4	1	3	9	2	4	2

■ Key Standard Topic □ Non-Key Standard Topic

It is important to understand that California started from “scratch” in synthesizing their new curriculum:

- Hired a group of Stanford math professors to take responsibility
- Obtained and studied the written curriculum for the A+ countries of Singapore and Japan, as well as a European country with a proud and lengthy tradition in math and science (Hungary)
- Carefully aligned their new curriculum to that of the reference countries, while writing the curriculum (standards) and the surrounding framework document to be suitable for North American students, teachers and parents.
- Kept carefully in mind the principals described in the previous paragraph (focussed, coherent, demanding in grades 6-8, not overly repetitive)
- Created a two-level system using the Key Standard approach to identify a core curriculum which every student must be taught, while providing plenty of extra standards for the bright students (a novel priority system).

What California did **not** do was:

- Attempt to band-aid their old NCTM-based 1990’s standards
- Attempt to modify an existing NSF-commercial curriculum such as TERC-CPM

Some schools and school districts aggressively adopted the new curriculum, bought matching textbooks and began teaching it in the ‘98-’99 school year.

Others stuck with the old U.S. national curriculums (based on the NCTM 1989 Standards).

All 2nd to 11th grade California students (97%) were tested using the nationally normed SAT-9 test, every year for the 5 year period 1998-2002 [4]. California switched to another national test for the year 2003.

Cohorts of 2nd to 6th grade students were analyzed for this period, as the best indication of learning built on previous learning (Figure 6).

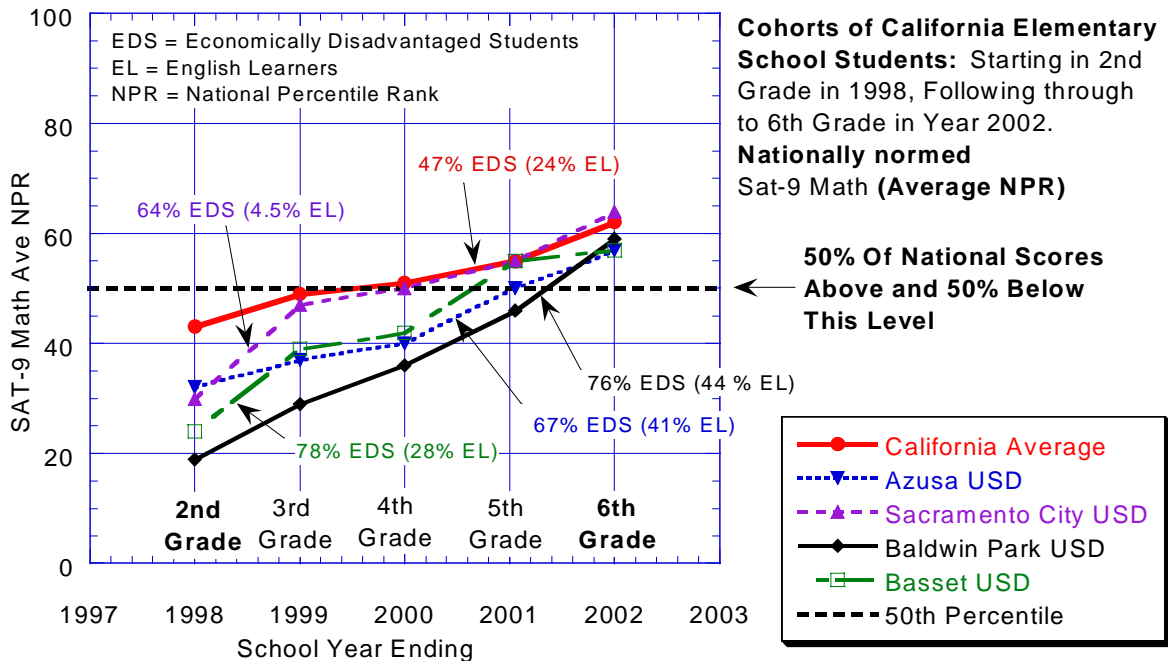


Figure 6 - State of California & Four Large Early Adoption Urban School Districts: Showing the stunning gains made by four large urban school districts relative to the California average. All these districts aggressively implemented and taught to the new California Math Standards starting in 1998, and purchased and used the California adopted Saxon Math Elementary School textbooks. All came close to the California average by 2002 despite starting as much as 24 percentile points below the average. All these districts had far higher ratios of Economically Disadvantaged Students than the California average of 47 %, and two had far higher percentages of English Learners.

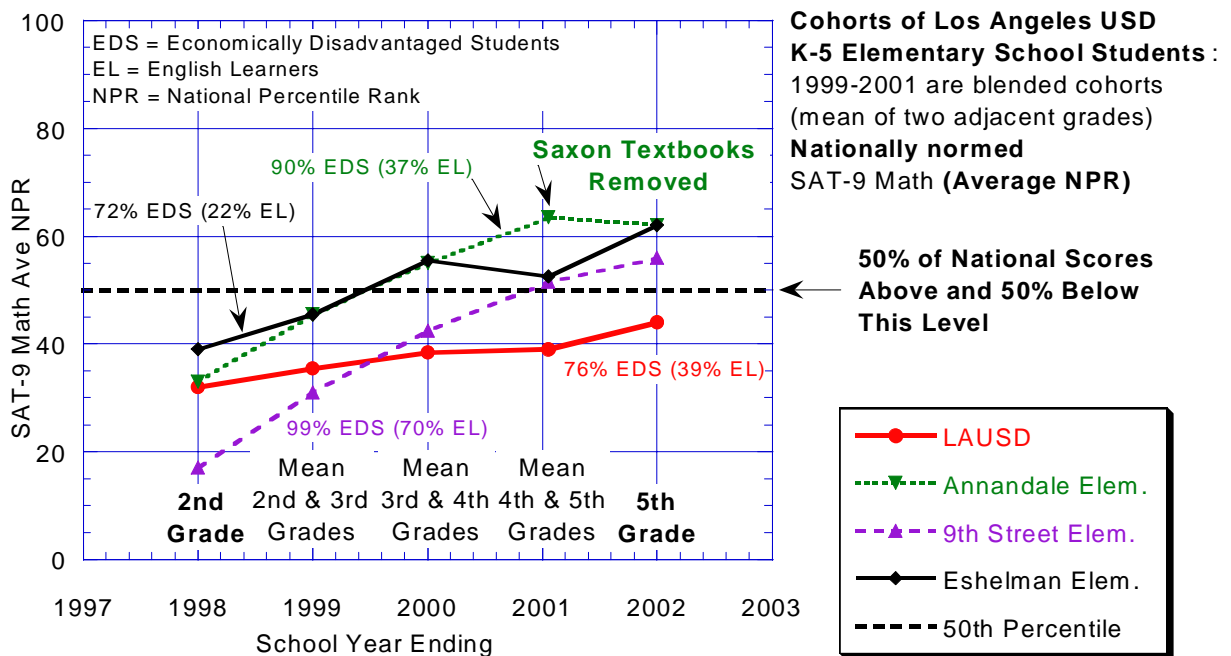


Figure 7 - Three Early Adoption Pilot Schools in the Los Angeles USD: Showing the stunning gains made by three Los Angeles USD inner city elementary schools relative to the LAUSD average. These three schools also aggressively implemented and taught to the new California Math Standards starting in 1998, and were allowed to use the Saxon Math textbooks as part of a pilot test program. 9th Street Elementary clearly had the best improvement record, raising it's score by 27 percentile points relative to the LAUSD average NPR. Notice 9th Street Elementary has 99 % EDS and 70 % English learners.

California statewide average shows excellent improvement, increasing from well below the “national average rank” at a 43 National Percentile Rank (NPR) in 1998 to well above at a 62 percentile rank in 2002:

- This was accomplished with a demographic of 47% economically disadvantaged students (EDS) and 24% English learners.
- It was accomplished with little help from the districts refusing to adopt the new standards (see cases of LA and San Diego below).

Four early adoption school districts showed stunning performance improvements during that same period:

- Baldwin Park USD improved by 40 NPR points, even with 76% economically disadvantaged students and 44% English learners.
- Sacramento City USD went from a percentile rank of 30 up to a rank of 64.
- Anecdotal reports of teachers thrilled with new textbooks, curriculum.

Data from districts which did not adopt the new curriculum are available as a research reference (a “control”). There are two such major school districts, totaling over 640,000 tested students:

- The Los Angeles USD refused to change, but allowed 3 pilot schools to become aggressive early adopters (Figure 7).
- The LAUSD performance is seen to be virtually flat.
- The champion pilot school is Ninth Street Elementary, with a gain of 39 percentile points, even though it had 99% EDS and 70% English learners.
- The other two pilot schools showed comparable gains, reaching above 60 National Percentile Rank.

San Diego City is the second major district refusing to adopt (Figure 8):

- San Diego chose to aggressively install a new NCTM-based curriculum in 1997, modeled after New York City District #2.
- Made massive investments in professional development, high quality teaching materials.
- Performance is seen to be virtually flat over the same period, as compared with the early adoption large districts described in Figure 6.
- An independent evaluation noted continued teacher dissatisfaction.

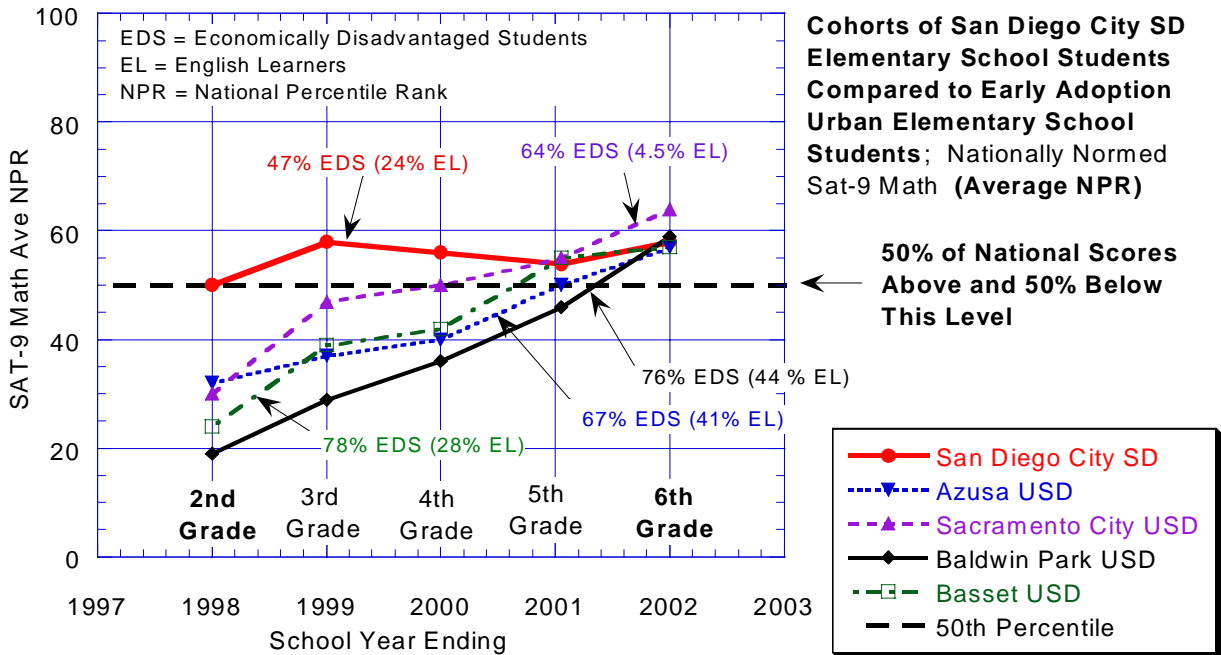


Figure 8 - San Diego City School District Elementary Schools Compared to Four Early-Adoption Urban Elementary School Districts: The total number of tested students in the early-adoption districts was 67,143, comparable to the SDC district. The average ratio of economically disadvantaged students was 68 % for these early-adoption districts, well above the San Diego ratio of 56 %.

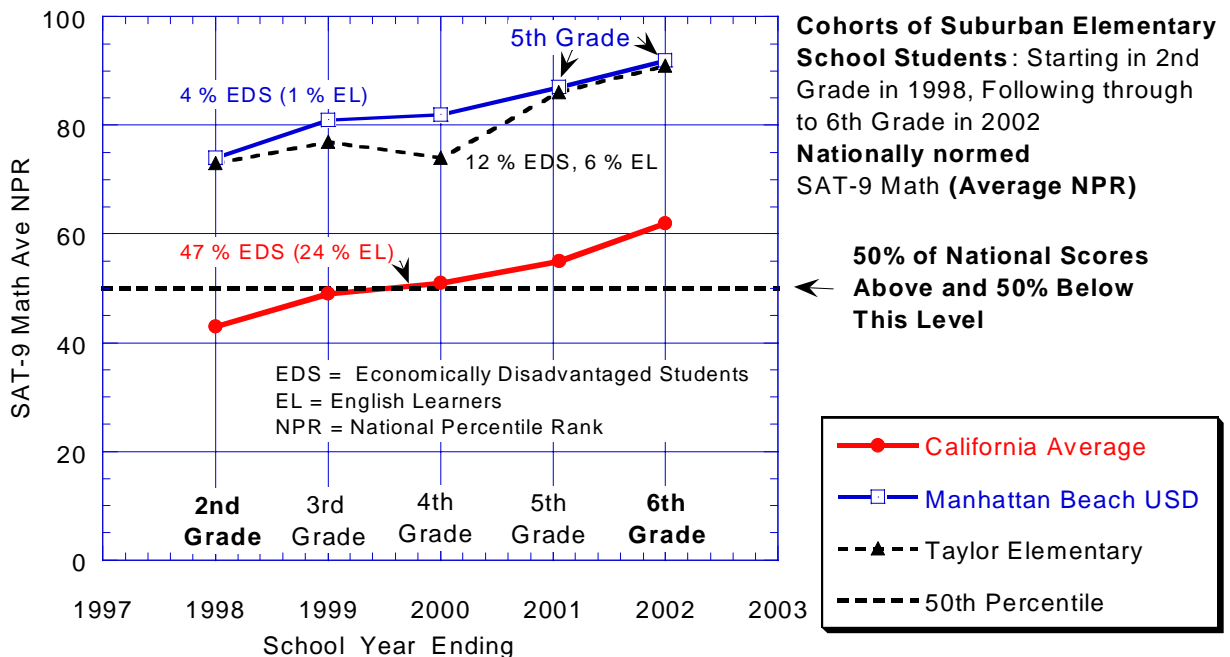


Figure 9- An Early Adoption Suburban School District and a School: Showing gains made by a district and a school with much lower percentages of Economically Disadvantaged Students than the California average of 47 %. Both aggressively implemented and taught to the new California Math Standard in 1998, and bought the Saxon Math textbooks for their elementary schools. Their performance was remarkable.

Results were astronomical at early adoption suburban public schools with low EDS (Figure 9), including Manhattan Beach USD and Taylor Elementary. Both reached above 90 average National Percentile Rank by 2002 (Figure 9), showing this improvement is not at the expense of traditionally high-performing students, corresponding to the experience of the A+ countries.

Summary and Conclusions, TIMSS Curriculum Study

Major conclusions from the TIMSS experimental research were:

- “the curriculum itself - what is taught - makes a huge difference”
- It is not primarily a matter of demographics nor of other non-school issues

Four characteristics of a curriculum were found to be important as compared to an A+ curriculum (expressed here in their negative form as found with the U.S. curriculum data):

- *Not focused* (too many topics, too little depth)
- *Highly repetitive* (topics introduced too early, endlessly repeated)
- *Incoherent* (not presented in logical, step-by-step order)
- *Not very demanding* (especially in middle school years)

The TIMSS method of quantitative evaluation of a curriculum using a “topics” analysis is a powerful analytic tool. It helped identify flaws in the U.S. curriculum, and gave information on the first three characteristics above.

The difference in the number of topics is usually the most dramatic result of a topics analysis. It must be emphasized, however, that the simple act of reducing the number of topics does not make an A+ curriculum.

It is also crucial to evaluate the fourth characteristic, *demanding in the middle grades* in relation to *coherence*. Algebra is a good example:

- High school math is now very algebra-intensive
- A graduate of B.C. 8th - 9th grade math must be proficient in algebra

A graduate of the 7th grade must be well prepared for 8th grade math, and must solve equations in one unknown, rate word problems, exponents, etc.

- Such a 7th grade “pre-algebra” set of topics would qualify as a *demanding* middle grade curriculum

In order to qualify as *coherent*, the topics leading up to 7th grade “pre-algebra” must be introduced in a logical, step-by-step manner.

- In California, this means algebra without symbols in the 2nd grade, and with simple symbolic algebra introduced in the 4th grade
- It also turns out that algebra is easy to learn in little steps when closely integrated with adding, subtracting, multiplying, dividing, fractions, etc.
- Thus a *coherent* curriculum is often one which is easy to learn.

The math standards for Singapore, Japan and California are all available in English to aid in understanding the level of rigor in an A+ country or in a curriculum aligned with an A+ country.

All the above constitute excellent methods which can be used to evaluate any existing or proposed curriculum.

Summary and Conclusions, California Curriculum Study

An A+ curriculum has been successfully tested in California over the 5 year period 1998-2002

Statewide average student improvement in elementary schools was 19 percentile (NPR) points, including 47% EDS and 24% English learners.

- This statewide improvement was achieved in the face of major school districts which refused to adopt the new curriculum and recorded feeble test score improvement

Urban early-adoption school districts showed improvements as high as 40 NPR points, even with higher percentages of disadvantaged students than the California average.

There are two excellent California “control” groups composing over 640,000 students, each refusing to adopt the 1997 curriculum.

The Los Angeles USD refused to adopt the new curriculum, and their improvement was only 10 percentile points.

- This dramatically contrasts with 3 early adoption pilot schools within the LAUSD which had improvement of 39, 29 and 23 percentile points.
- The champion LA early adoption school had 99% EDS, and 70% English learners.

The San Diego City School District also refused to adopt the new curriculum, and their improvement was even worse at 8 percentile points.

- This poor result may be compared to the outstanding performance of the four urban early adoption school districts described earlier, where those districts have a total size comparable to San Diego and with more disadvantaged students.
- San Diego improvement was less than half the improvement for the state as a whole.

An early adoption suburban district and a school showed astronomical performance at above 90 percentile for the average student, demonstrating this type of curriculum also helps the traditionally bright students.

Overall Conclusions

All the above sensational results come from what is certainly two of the most expensive and well controlled education experiments in history [5]

It has been clearly shown that a North American version of a world-class elementary school curriculum, such as that of California, can provide the average or economically disadvantaged student with a quality math education and an excellent preparation for high school math, with emphasis on pre-algebra.

This curriculum also provides the bright student with a superior preparation for high school and college.

The education experiment underlying these results comes close to duplicating the features of the best sort of hard science experiment, in that it was successfully run both forwards and backwards:

- The essential characteristics were identified in the first experiment.
- In the second experiment, these characteristics were synthesized, and the synthesized characteristics produced the same result as the original characteristics.
- In both cases, the sample size was huge.

Notice none of these results depends on any theory of education nor theory of learning.

They originate entirely from a U.S. Congress mandate to “find out how math and science teaching in the U.S. compares to the rest of the world”, and from a reform movement led by California parents and academics and supported by non-partisan cooperation between a Republican governor and a Democratic Legislature.

Sources

1. The Curriculum Analysis was done by the U.S. National Research Center for the TIMSS at Michigan State University, School of Education, William Schmidt director, and is based on data obtained during the 1995 TIMSS. A summary paper describing the status of the curriculum research as of 2002 was published in the American Educator, Summer 2002, and is the basis for the material presented here.

http://www.aft.org/american_educator/summer2002/curriculum.pdf

The American Educator paper is also based on research first described in three earlier books by Schmidt, et. al., “Why Schools Matter”, “Facing the Consequences”, and “A Splintered Vision”. The general website for the Michigan State TIMSS Center is:

<http://ustimss.msu.edu/>

A variety of more recent TIMSS material can be found on the U.S. Department of Education website, including comparisons of the 1995 and 1999 results. Search “TIMSS”. <http://www.ed.gov/index.jhtml>

2. The country rankings for the 1995 TIMSS can be found at the Boston College TIMSS International Study Center archives, <http://timss.bc.edu/timss1/highlights.html>

3. The entire California curriculum can be found in “Mathematics Framework for California Public Schools, 2000 Revised Edition”

<http://www.cde.ca.gov/re/pn/fd/documents/mathematics-frame.pdf>

4. All the California test and demographic data presented here is easily available at the California Department of Education web site, and is organized by year, school and grade: <http://star.cde.ca.gov/> (sometimes slow because of website reorganization, but it does work)
5. General: William Hook has written a paper further summarizing the Michigan State University TIMSS conclusions, and adding an analysis of the California curriculum using the Michigan protocol.

Dr. Wayne Bishop, a math professor at Cal State University, LA and William Hook have written a research paper summarizing and analyzing the California test data for the 5 year period 1998-2002.

This presentation is based specifically on the two papers by Hook and Bishop, and are available from William Hook, whook@uvic.ca They are:

- Bishop, Wayne and William Hook, “Urban Elementary Schools in California Show Stunning Improvement in SAT-9 Test Scores over Initial Four Year period of New Math Standards”, January 15, 2004
- Hook, William, “ ‘Curriculum Makes a Huge Difference’ – A Summary of Conclusions from the Trends in International Mathematics Study (TIMSS) with California Data Added”, March 5, 2004

Appendix

Mathematics Topics: <http://currmap.ncrel.org/mathTopicsList.htm>

Numbers

Whole Numbers

Whole Numbers: Meaning

Whole Numbers: Operations

Whole Numbers: Properties of operations

Fraction and Decimals

Common fractions

Decimal fractions

Relationships of common and decimal fractions

Percentages

Properties of common and decimal fractions

- Integer, Rational and Real Numbers
 - Negative numbers, integers, and their properties
 - Rational numbers and their properties
 - Real numbers, their subsets, and their properties
- Other Numbers and Number Concepts
 - Binary arithmetic and/or other number bases
 - Exponents, roots, and radicals
 - Complex numbers and their properties
 - Number theory
 - Counting
- Estimation and Number Sense
 - Estimating quantity and size
 - Rounding and significant figures
 - Estimating computations
 - Exponents and orders of magnitude
- Measurement**
 - Measurement and Units
 - Perimeter, area, and volume
 - Estimation and errors
- Geometry: Position, Visualization, and Shape**
 - Two-dimensional geometry: coordinate geometry
 - Two-dimensional geometry: basic
 - Two-dimensional geometry: polygons and circles
 - Three-dimensional geometry
 - Vectors
- Geometry: Symmetry, Congruence, and Similarity**
 - Transformations
 - Congruence and similarity
 - Constructions using straight-edge and compass
- Proportionality**
 - Proportionality concepts
 - Proportionality problems
 - Slope and trigonometry
 - Linear interpolation and extrapolation
- Functions, Relations, and Equations**
 - Patterns, relations, and functions
 - Equations and formulas
- Data Representation, Probability, and Statistics**
 - Data representation and analysis
 - Uncertainty and probability
- Elementary Analysis**
 - Infinite processes
 - Change (growth and decay, differentiation)
- Validation and Structure**
 - Validation and justification
 - Structuring and abstracting
 - Other content

